

Dynamics



Newton's Second Law of Motion

- The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

$$\Sigma F = ma$$

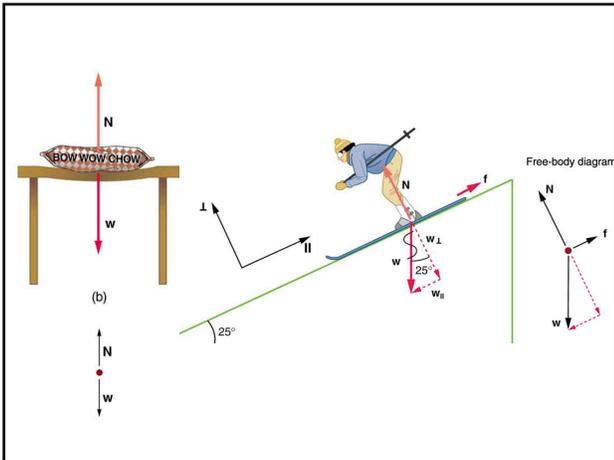
Translational Equilibrium

- An object is said to be in translational equilibrium if and only if the net force on the system is zero.
 - The vector sum of all the forces acting on the system is zero.

$$\begin{aligned}\Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma F_z &= 0\end{aligned}$$

Normal Force

- When an object is sitting on a surface the surface must support the load by exerting an upwards force equal to the weight.
- If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a normal force.



Friction

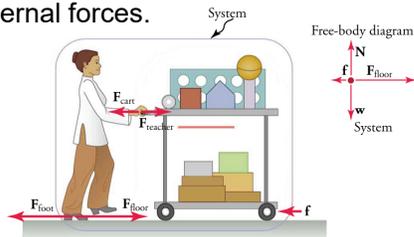
- Friction is an external force that acts opposite to the direction of relative motion or to prevent slipping.
- The magnitude of the force of friction is proportional to the normal force.

$$F_f = \mu F_N$$

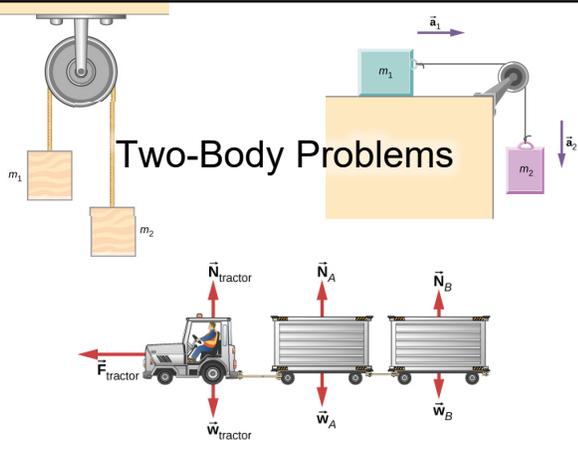
μ is the coefficient of friction

System

- A system is defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces.

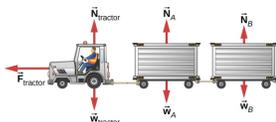


Two-Body Problems

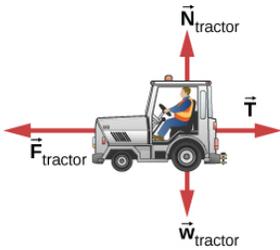


Example 1

A baggage tractor pulls luggage carts from an airplane. The tractor has mass 650.0 kg, while cart A has mass 250.0 kg and cart B has mass 150.0 kg. A driving force of 820.0 N acts for 3.00 s accelerating the system from rest. What is the horizontal force acting on the connecting cable between the tractor and cart A at this instant?



To calculate the tension between the tractor and cart A, we need to treat the tractor as a system.



$$\Sigma F = ma$$

$$F_{tractor} - T = ma$$

$$T = F_{tractor} - ma$$

To calculate the acceleration, we need to look at the system of the tractor and the baggage cars.



$$\Sigma F = ma$$

$$F_{tractor} = (m_{tractor} + m_A + m_B)a$$

$$a = \frac{F_{tractor}}{(m_{tractor} + m_A + m_B)}$$

$$a = \frac{820}{(850 + 250 + 150)} = 0.67 \text{ m/s}^2$$

Now we can solve for the tension between the tractor and cart A.

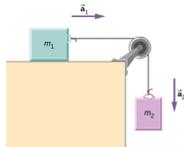
$$T = F_{tractor} - ma$$

$$T = 820 - (850)(0.67)$$

$$T = 250 \text{ N}$$

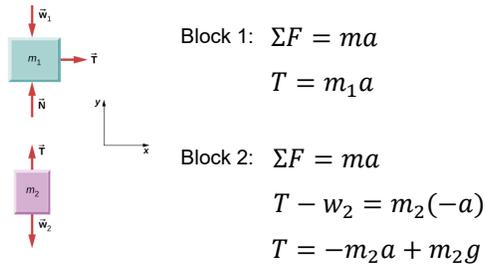
Example 2

A block on a frictionless, horizontal surface is pulled by a light string that passes over a frictionless and massless pulley. The other end of the string is connected to another block. The mass of block 1 is 1.0 kg and the mass of block 2 is 2.0 kg. Calculate the tension in the string joining the blocks.

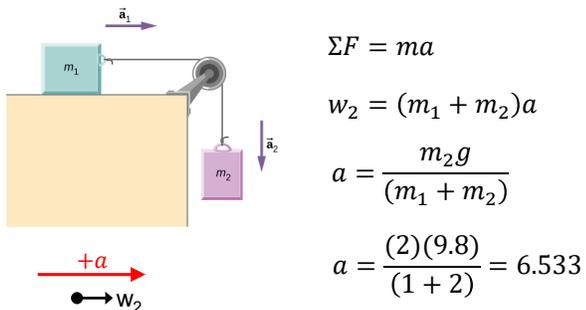


To calculate the tension in the string we need to treat either block 1 or block 2 as a system.

Note: Block 1 accelerates in the positive x direction. Block 2 accelerates in the negative y direction.



To calculate the acceleration, we need to consider the system of both blocks.



Now we can calculate the tension in the string.

Note: We can use either block 1 or block 2.

Block 1

Block 2

$$T = m_1 a$$

$$T = -m_2 a + m_2 g$$

$$T = (1)(6.533)$$

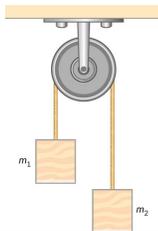
$$T = -(2)(6.533) + (2)(9.8)$$

$$T = 6.5 \text{ N}$$

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Example 3

An Atwood machine consists of a rope running over a pulley, with two objects of different mass attached. If $m_1 = 2.00 \text{ kg}$ and $m_2 = 4.00 \text{ kg}$, what is the tension in the string?



To calculate the tension in the string we need to treat either block 1 or block 2 as a system.

Block 1

Block 2

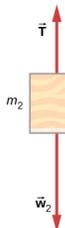
$$\Sigma F = ma$$



$$T - w_1 = m_1 a$$

$$T = m_1 a + w_1$$

$$T = m_1 a + m_1 g$$



$$\Sigma F = ma$$

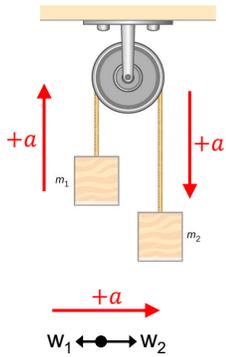
$$T - w_2 = m_2(-a)$$

$$T = -m_2 a + w_2$$

$$T = -m_2 a + m_2 g$$

Note: the blocks accelerate in opposite directions

To calculate the acceleration, we need to consider the system of both blocks.



$$\Sigma F = ma$$

$$-w_1 + w_2 = (m_1 + m_2)a$$

$$a = \frac{-m_1g + m_2g}{(m_1 + m_2)}$$

$$a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$$

$$a = \frac{(4 - 2)(9.8)}{(2 + 4)} = 3.27 \text{ m/s}^2$$

Now we can calculate the tension in the string.

Note: We can use either block 1 or block 2.

Block 1

Block 2

$$T = m_1a + m_1g$$

$$T = -m_2a + m_2g$$

$$T = m_1(a + g)$$

$$T = m_2(-a + g)$$

$$T = 2(3.27 + 9.8)$$

$$T = 4(-3.27 + 9.8)$$

$$T = 26.1 \text{ N}$$

$$T = 26.1 \text{ N}$$
